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(***UGC recognized, NAAC Re-accredited 'B++' grade state university***)

"A STUDY ON STOCHASTIC PROGRAMMING PROBLEM OPTIMIZATION"

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In the Subject

Statistics faculty of science and technology

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"A Study on Stochastic Programming Problem Optimization"

1. Introduction:

Stochastic programming (SP) is a mathematical framework model and is used to solve real life problems that involve uncertainty. SP extends traditional optimization by considering this uncertainty in the decision-making process. In traditional optimization, the parameters of the problem are assumed to be fixed and known with certainty. It involves incorporating probabilistic information about uncertain parameters into the optimization problem, and then seeking solutions that are robust and effective across various possible outcomes of the uncertain events. It is used to optimize decisions in a way that takes into account the trade-offs between achieving good outcomes in all possible scenarios versus focusing on the most likely outcomes. However, many real-world problems, such as financial decisions, supply chain management, and resource allocation, are influenced by random or uncertain factors like market fluctuations, demand variability, and other sources of unpredictability. Various techniques are used, such as Monte Carlo simulation, scenario generation, and stochastic approximation, to approximate or solve these problems. These methods help decision-makers make more informed and robust choices when dealing with uncertain environments. **[6]**

It has grown into a robust field of study, with continuous advancements in theoretical foundations, algorithm development, and applications. Researchers and practitioners in various domains, including finance, operations research, logistics, energy, and environmental management, have embraced SP to optimize complex decision problems in uncertain and dynamic environments.[6 ,30]

Dantzig [9] played pivotal roles in the development of SP. Their work introduced the notion of optimizing under uncertainty, paving the way for an essential branch of mathematical optimization that remains relevant and vital in addressing real-world decision-making challenges to these days **[8].** There are two main categories of stochastic programming:

1.1 Two-Stage Stochastic Programming [6]:

In this approach, decisions are made in two stages. The first stage involves asking decisions that are not influenced by uncertainty (e.g., determining production levels or investment amounts). In the second stage, after the realization of uncertain events, decisions are adjusted to achieve the best possible outcomes in light of the actual outcomes of those events.

The classical two-stage stochastic linear program with fixed recourse [6] is the problem of finding

$$\begin{gathered} Min\, z = c^T x + E\xi\,[min\, q(\omega)\, T\, y(\omega)] \\ s.t. Ax = b, \\ T(\omega)x + W\, y(\omega) = h(\omega), \\ x \geq 0\,, y(\omega) \geq 0 \end{gathered}$$

1.2 Multi-Stage Stochastic Programming [6]:

It expands the two-stage concept to more than two stages. It's suitable for problems where the decision process unfolds over multiple time periods and decisions need to be adjusted as new information becomes available.

The multistage stochastic linear program with fixed recourse can be written as;

$$Min\, z = c^1x^1 + E_{\xi^2}[min\, c^2(\omega)x^2(\omega^2) + \cdots + E\xi H\, [min\, c^H(\omega)x^H(\omega^H)] \cdots]$$

$$s.t. W^1x^1 = h^1,$$

$$T^1(\omega)x^{H-1} + W^2x^2(\omega^2) = h^2(\omega),$$

$$\vdots$$

$$T^{H-1}(\omega)x^{H-1}(\omega^{H-1}) + W^Hx^H(\omega^H) = h^H(\omega),$$

$$x^1 \geq 0;\ x^t(\omega^t) \geq 0, t = 2, \ldots, H;$$

Where, $c^1$is a known vector in$\Re^{n_1}$,

h^1 is a known vector in,$\Re^{m_1}\xi^t(\omega)^t$= $(c^t(\omega)^t, h^t(\omega)^t$,

$T_1^{t-1}(\omega)$,...,$T_{m_t}^{t-1}(\omega)$ is a random N_t-vector defined on (Ω, Σ^t, P) (where $\Sigma^t \subset \Sigma^{t+1}$),

$\forall\ t = 2, \ldots, H$, and each W^t is a known $m_t \times n_t$ matrix.

The decisions x depend on the history up to time t, which we indicate by ω^t. We also suppose that $\Sigma^t$is the support ξ^t.

1.3 Stochastic modeling [24]:

It used to represent, analyze and predict systems or processes influenced by randomness or uncertainty. It provides insights into how such systems behave over time or under various conditions. The random variables are usually constrained by historical data such as part market returns. The Monte Carlo simulation is a tool to select data for constructing stochastic modeling which use in portfolio evaluation, multiple simulations of the performance of the portfolio are done based on the probability distributions of the individual stock returns. A statistical analysis of the results can help to determine the probability that the portfolio with provide the desired performance. It is a technique of presenting data or predicting outcomes that taken into account a certain degree of randomness, or unpredictability. The Insurance agency depends greatly on stochastic modeling for predicting the future conditions of company balance sheets, since these may depend on unpredictable events resulting in the paying of the claims. Many other industries and fields of study can benefit from stochastic modeling, such as Statistics, Operation Research, stock investing, biology, economics, risk analysis and management etc.

1.4 Basic Stochastic Programming Models [18]:

The anticipative and the adaptive models are special cases of stochastic programs. The combination of them makes a recursive model which is widely applied in financial field.

1.4.1 Anticipative models: Anticipative model is also referred to as static model, for which the decision does not depend in any way on future observations of the environment. The prudent planning has to take into account all possible future realizations since there is no opportunity to adapt decisions later on, which may lead to overly conservative decisions.

In anticipative models' feasibility is expressed in terms of probabilistic (or chance) constraints. For example, a reliability level a, where $0 < \alpha \leq 1$, is specified and constraints are expressed in the

$$P\{\omega/(x,\omega) = 0, j = 1,2,\ldots,n\} \geq \alpha$$

Where x is the m-dimensional vector of decision variables and $f_j: R^m \times \Omega \to \text{R}, j = 1,2,\ldots,n$ the objective function may also be of a reliability type, such as $P\{\omega \,|f_0(x,\omega) \leq \Upsilon\}$ where $f_0: \mathbb{R} \times \Omega \to \mathbb{R} \cup \{+\infty\}$ and Υ is a constant.

An anticipative model selects a policy that meets desirable characteristics of the constraints and the objective function. In the example above, it is desirable that the probability of a constraint violation is less than the pre-specified threshold value 1-α. The precise value of a depends on the application at hand, the cost of constraint violation, and other similar considerations.

1.4.2 Adaptive models:

In an adaptive model, information related to the uncertainty becomes partially available before decision making, so optimization takes place in a learning environment, which is the essential difference with an adaptive model.

Let $\mathcal{A}$ be the collection of all the relevant information available through observation, which is a subfield of all possible events. The decision x depends on the events that can be observed, and x is termed $\mathcal{A}$-adapted of $\mathcal{A}$-measurable. An adaptive stochastic program can be formulated as:

$$\text{Minimize } E\,[f_0(x(\omega),\omega)|\,\mathcal{A}]$$

$$\text{Subject to } E\left[f_j,(x(\omega),\omega|\mathcal{A})\right] = 0 \quad j = 1,2,\ldots,n$$

$$x(\omega)\,\epsilon\ \text{X} \qquad \text{almost surely}$$

The mapping $x: \Omega \to$ Xis such that $x(\omega)$ is $\mathcal{A}$-measurable. This problem can be addressed by solving for every ω the following deterministic programs:

$$\text{Minimize} E[f_0(x,\ \cdot)|\mathcal{A}](\omega)$$

$$\text{Subject } to \quad E\left[f_j(x,\ \cdot)\middle|\mathcal{A}\right](\omega) = 0 \quad j = 1,2,\ldots,n$$

$$x\,\epsilon\,\text{X}$$

The two extreme cases, complete information and no information at all, deserve special mention. The latter reduces the model to the anticipative form while the former is known as distribution model, which characterizes the distribution of the optimal objective value. However, the most interesting and valuable situation arises when partial information is available, which is what we will discuss below.

1.4.3 Recourse models:

The recursive model combines the former two models in a common mathematical framework, which seeks a policy that not only anticipates future observations but also takes into account temporarily available information to make recourse decisions.

For example, a portfolio manager considers both future movements of stock prices (anticipation) as well as rebalancing the portfolio positions as prices change (adaptation).

The two-stage stochastic programming problem with recourse can be written as follows:

$$\text{Minimize } f(x) + E[Q(x, \omega)]$$

$$\text{Subject to } Ax = b$$

$$x \in \mathbb{R}_{+}^{m_0}$$

Where, x is the first-stage anticipative decisions, which is made before the random variables are observed, and$Q(x, \omega)$ is the optimal value, for any given Ω, of the following nonlinear program

$$\text{Minimize } q(y, \omega)$$

$$\text{Subject to } W(\omega)y = h(\omega) - T(\omega)x$$

$$y \in \mathbb{R}_{+}^{m_1}$$

Where, y is the second-stage adaptive decision, which depends on the realization of the first-stage random vector. $q(y, \omega)$Denotes the second-stage cost function, and $\{T(\omega), W(\omega), h(\omega) | \omega \in \Omega\}$ are model parameters with reasonable dimensions. Those parameters are functions of the random vector ω and are, therefore, random parameters. T is the technology matrix containing the technology coefficients that convert the first-stage decision into resources for the second-stage problem. *W* is the recourse matrix and h is the second-stage resource vector. Generally the two-stage recourse model can be formulated as follows:

$$Minimize \quad f(x) + E\left[\min_{y \in \mathbb{R}_{+}^{m_0}} \{(q(y, \omega)\, T(\omega)x + W(\omega) - h(\omega)\}\right]$$

$$\text{Subject to} Ax = b \quad (*)$$

$$x \in \mathbb{R}_{+}^{m_0}$$

1.4.4 Multistage models:

The recourse problem is not restricted to the two-stage formulation. It is possible that observations are made at Tdifferent stages and are captured in the information sets $\{A_t\}_{t=1}^{T}$with$A_1 \subset A_2 \ldots, \subset A_T$. Stages correspond to time instances when some information is revealed and a decision can be made. (Where, T is a time index, while $T(\omega)$are matrices.)

A multistage stochastic program with recourse will have a recourse problem at stage τconditioned on the information provided by $A_t$which includes all information provided by the information sets A_t, for $t = 1, 2, \ldots, \tau$. The program also anticipates the information in $A_t, for\ t = \tau + 1, \ldots T$.

Let, the random vector ω have support $\Omega=\Omega_1 \times \Omega_2 \times \ldots \times \Omega_T$, which is the product set of all individual support sets $\Omega_t, t = 1,2, \ldots, T$. w is written componentwise as $\omega = (\omega_1, \omega_2, \ldots, \omega_T)$De-note the first-stage variable vector by y_0. For each stage $t = 1,2, \ldots, .T$, define the recourse variable vector $y_t \in \mathbb{R}^{m_t}$,the random cost function $q_t(y_t, \omega_t)$, and the random parameters $\{T_t(\omega_t), W_t, h_t(\omega_t), h_t(\omega_t) | \omega_t \in \Omega_t\}$.

The multistage program, which extends the two-stage model (b), is formulated as the following nested optimization problem and represented as;

$$Minimize \quad f(y_0) + E[\min_{y_1 \epsilon\, \mathbb{R}_+^{m_1}} q_1(y_1, \omega_1) \dots E\,[\min_{y_1 \epsilon\, \mathbb{R}_+^{m_T}} q_T(y_T, \omega_T)] \dots]$$

$$subject\ to \quad T_1(\omega_1) + W_1(\omega_1) y_1 = h^1(\omega_1),$$

$$\vdots$$

$$T_T(\omega_T) y_{T-1} + W_T(\omega_T) y_T = h_k(\omega_T),$$

$$y_0 \epsilon\, \mathbb{R}_+^{m_0}$$

For the case of discrete and finitely distributed probability distributions it is again possible to formulate the multistage model into a deterministic equivalent large-scale nonlinear program.

1.5 Stochastic Optimization [24]:

Many concrete problems from different fields can be modeled using mathematical programs. Mathematical programming refers to mathematical models used to solve problems and make optimal decisions. It can seek to maximize or minimize an objective function, often subject to a set of constraints. More precisely, the formulation of an optimization problem involves:

1. Selecting optimization variables
2. Defining the objective function
3. Identifying the set of constraints

The general model can be represented as:

$$min\,\{\, f(x)\ /g(x, a)\ \geq\ 0\}$$

Where, f is the objective function,

x is a decision vector,

a is a vector of model parameters, and

g is the constraint mapping.

Model parameters can be unknown or uncertain and may imply randomness. Historically, uncertainty has been considered in mathematical models for decision-making. Many approaches have been proposed to address this issue. One of the widely used approaches is stochastic programs, as they have proved their flexibility and usefulness in different areas of science. In fact, stochastic models rely on solid mathematical foundations, probability theory, and stochastic processes. [6,30]

Stochastic optimization refers to a collection of methods for minimizing or maximizing an objective function when randomness is present. Randomness usually enters the problem through the objective function and/or the constraint set.

Over the past few decades, this field has been an active area of research. With the increase of theoretical and algorithmic developments, stochastic programming has been applied to a wide variety of problems, including water resources planning and management, production planning, and financial problems. It plays an important role in the analysis, design, and performance of modern systems.

An important number of optimization problems involving uncertainty occur if the constraints g depend on a stochastic parameter ξ, such as the system in might be:

$$Min\{f(x) \mid g(x,\xi) \geq 0\}$$

Here, ξ is a k-dimensional random variable defined on some probability space (Ω, F, P).

To deal with uncertainty and arrive at a relevant and implementable form of the constraints, the dependence of g on specific outcomes of ξ has to be removed. The most prominent approaches to do so are:

1. The expected value approach
2. The approach by chance constraints: types of chance constraints given as:

- **Joint chance constraints:** It ensures that all constraints in a system are satisfied simultaneously with a given probability threshold. [7]
- **Single chance constraints:** It ensures that a specific constraints is satisfied with a given probability threshold. [7]

2. Review of literature and development in the subject:

SP models have been proposed and well-studied since late 1950s by Dantzig [8, 9], Beale[10], Charnes and Cooper[7] and others. Levkovitz and Mitra [17], Mulvey et al. [22] they proposed a stochastic view to replace the deterministic one, where the unknown coefficients or parameters are random with assumed probability distribution that is independent of the decision variables. Li [18] in their study they survey the SP models developed to deal with financial optimization problems. Juan Marquez et al. [12] in their study they are performs a literature review on agricultural water allocation based on MSP, identifying crop yield as the principal farmers' benefits of proper water allocation.

Konicz, et al. [2] combined stochastic programming and optimal control approach to personal finance and pension. They combined a dynamic programming approach (stochastic optimal control) with a multi-stage stochastic programming approach (MSP) in order to solve various problems in personal finance and pensions. Both optimization methods are integrated into one MSP formulation. Kawtar and Driss [11] Focused on SP and covers some of the theoretical foundations. Also, focuses on recent advances in agriculture as an area where SP is applicable. Torkamani and Hardaker [13] found that risk aversion may play an important role in farmer's behavior. Farmers were predicted to sow higher risk cash crops with higher payoffs only when their coefficient of absolute risk aversion was low. Anderson and Dillon [3] argued, may be rational for the individual farmer but results in output levels and product combinations that from society's point of view are inefficient. However, farmers' risk-taking behavior may be influenced by improved education and promoting risk-mitigating strategies such as more reliable technologies or crop insurance. These strategies might lead farmers to increase agricultural production with consequent benefits to social welfare. Mirzaei, et al. [1] studied the management of water resources demand in the agricultural sector due to the uncertainty of those resources is considered by combining the issues of surface water resources release management in irrigation networks, and crop pattern optimization under seasonal fluctuations of surface water resources in irrigation networks. Attention has also been paid to the intensity of cultivation and consumption of groundwater resources as an alternative to surface water and finally they conclude that the present study can help adoption of desired policies for irrigation water management through the proper release of these resources. Apl and Hauer [14] provides an overview of the discrete stochastic programming model. The conceptual basis of the model is presented and illustrated with numerical examples, and a summary of empirical applications of the technique is presented. The

purpose is to introduce discrete SP to those familiar with risk analysis and to provide a concise review of empirical applications. Li and Hu [25] proposed an MSP model for farm land irrigation management under uncertainty. They found the number of limitations that we may use for future research plan. The limitations: 1) the numerical results reported in the case study are the best feasible solution in a reasonable computational time. More efficient algorithms and heuristic solutions based on artificial intelligence and meta- heuristic methods such as Genetic Algorithms and Artificial Neural Networks should be investigated. 2) The case study only illustrates the model of a center pivot sprinkler system with almost homogeneous soil features. Other irrigation systems and land profiles could also be investigated. 3) They consider three sources of independent uncertainties, and more dependent stochastic factors can be considered in farm-level irrigation problems. Last but not least, this model focuses on a single-year profit maximization problem. In addition, as suggested by the reviewers, the evaluation of new irrigation system installations in a multi- year horizon considering water allocation policy limitations is another interesting research topic. Li, et al. [27] in their study they developed a hybrid fuzzy-stochastic programming method for planning water trading under uncertainties of randomness and fuzziness. The method can deal with recourse water allocation problems generated by randomness in water availability and, at the same time, tackle uncertainties expressed as fuzzy sets in the trading system. Ren et al. [26] developed an IMOSFP model to optimization allocation of irrigation water and land resources simultaneously, which has the characteristics with multiple objectives with conflicts under multiple uncertainties. The proposed model could deal with uncertainties expressed as fuzzy sets and randomness, and could offer a range of optimization schemes under different alpha-cut level of different risk probabilities P_i. Graveline, et al. [23] The future application of the joint Monte Carlo – linear programming method holds promise for investigating how individual farm parameters react to different scenarios. This innovative method offers a robust framework to analyze complex dynamics, potentially uncovering disparities and aiding targeted policy decisions for sustainable farming practices. Colvin and Maravelias [24] developed a multi-period SP formulation that accounts for the uncertainty in the outcome of clinical trials. Higle [16] presented an introduction to stochastic programming models and methodology at a level that is intended to be accessible to the breadth of members within the INFORMS community. Torres, et al. [27] presented a tutorial on the state-of-the-art software for the solution of two-stage (mixed-integer) linear stochastic programs and provides a list of software designed for this purpose. They provided a tutorial for beginners on linear and mixed-integer linear stochastic programming, introducing the fundamentals on solution methodologies with a focus on available software. They provided a general description of techniques based on Bender's decomposition and dual decomposition, which are the fundamentals of most known solvers designed specifically to tackle special problem structures. Lustig et al. [18] described an approach for modeling two-stage stochastic programs that yields a form suitable for interior point algorithms. Burdett et.al [28] introduces an approach with risk-averse sp to assess hospital capacity, significant advancement in healthcare operations research. Yin & Büyüktahtakın [29] introduced a multi-stage sp model to optimize resource allocation during an epidemic, considering uncertainty in disease progression and resource availability. Shapiro et al. [30] provided seminal work in the field of optimization under uncertainty and also provide a rigorous and in-depth exploration of the theoretical foundations and practical applications of sp.

3. The Scope of Study:

The SP is having dynamic and wide-ranging scope as this field continues to evolve with advances in mathematical optimization, computational techniques, and applications across various industries. Here are some potential areas of future research in SP:

1. Developing more efficient and scalable algorithms for solving complex stochastic programming problems.
2. Enhancing methods to model and characterize uncertainty is a crucial research direction.
3. Research could focus on advancing robust optimization techniques that ensure solutions remain feasible and perform well under a range of uncertain scenarios, without having to explicitly model a probability distribution.
4. This emerging area seeks to optimize decisions against a set of plausible probability distributions instead of a single known distribution. Exploring the integration of machine learning techniques with stochastic programming could lead to enhanced methods for predicting uncertain parameters, generating scenarios, and improving optimization outcomes.
5. Research could delve into more complex multi-stage and dynamic stochastic programming models, addressing the challenges of making decisions over multiple periods or stages while considering evolving uncertainties.
6. Investigating hybrid approaches that combine stochastic programming with other optimization techniques, such as integer programming, nonlinear programming, or game theory, could lead to novel solution strategies.
7. Developing real-time decision-making frameworks that can adapt to changing uncertainty levels and make optimal decisions on the fly is an emerging area of research.
8. As environmental concerns grow, researchers could focus on using stochastic programming to address sustainability challenges, such as optimizing resource usage and mitigating the impacts of climate change.
9. Recent events have highlighted the need for effective planning in healthcare systems. Future research could investigate stochastic programming applications in healthcare capacity planning, epidemic management, and medical resource allocation [28, 29].
10. Advancing methods to model and optimize complex systems, such as smart cities, transportation networks, and energy grids, under uncertainty is an area with significant potential [6].
11. Exploring new domains and interdisciplinary applications where stochastic programming can provide valuable insights and solutions, such as social sciences, economics, and policy-making.

Overall, SP is multifaceted, spanning a wide range of methodological, computational, and application-focused areas.

4. Applications of Stochastic Programming [6, 30]:

It has been applied to different areas and some of the specific areas where SP problems may occurred are listed below:

- Manufacturing production planning
- Manufacturing production capacity planning
- Electrical generation capacity planning
- Machine scheduling
- Freight scheduling
- Dairy farm expansion planning
- Macroeconomic modeling and planning
- Timber management
- Asset liability management

- Portfolio selection
- Traffic management
- Automobile dealership inventory management
- Lake level management

5. Objectives of the Research:

The primary objectives of this research are as follows:

1) To study stochastic programming algorithms.
2) To develop stochastic programming problem with different stages (two-stage and multi-stage)
3) To compare proposed model with existing model.
4) To compare the advantages and limitations of different modeling approaches in capturing real-world uncertainty.
5) To identify and analyze the diverse range of applications where stochastic programming plays a significant role.

During the period of research many new objectives may be developed which will be beneficial for the society

6. Methodology:

1) Collection of relevant literature from different institutional libraries in India.
2) Consultations with expertise on topic relevant to proposed work
3) The Mathematical approach will be undertaken for the model development.
4) Advanced Stochastic programming techniques will be used to solve the problems.
5) The developed model will be tested with real life scenarios.

7. Proposed work Plan/ Formulation and Structure of Study:

Year wise Plan of work and targets to be achieved

❖ **First Year**

1. To collect and study of review of literature of stochastic programming.
2. Collection and Study of different SPP algorithms.
3. Study of different Software packages.

❖ **Second Year**

1. Try to develop stochastic programming problem algorithms.
2. Detailed study and collaborative analysis of the results obtained.
3. The comparative study of the use of different stochastic models for the same problems

❖ **Third Year**

1. The publication of the findings of the research.
2. Publication of papers in various national and international journals and conferences.
3. Thesis writing.

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